If $q(x) = \int_{e^{3x}}^{5x} \cos t^2 \, dt$, find $q'(x)$. **SCORE: _____ / 4 PTS**

$$q(x) = \int_{e^{3x}}^{0} \cos t^2 \, dt + \int_0^{5x} \cos t^2 \, dt$$

$$= -\int_0^{e^{3x}} \cos t^2 \, dt + \int_0^{5x} \cos t^2 \, dt$$

$$q'(x) = -\frac{d}{dx}\int_0^{e^{3x}} \cos t^2 \, dt + \frac{d}{dx}\int_0^{5x} \cos t^2 \, dt$$

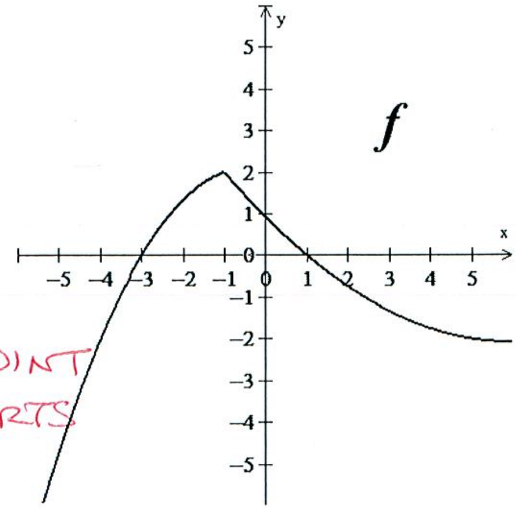
$$= -\frac{d}{d(e^{3x})}\int_0^{e^{3x}} \cos t^2 \, dt \cdot \frac{d(e^{3x})}{dx} + \frac{d}{d(5x)}\int_0^{5x} \cos t^2 \, dt \cdot \frac{d(5x)}{dx}$$

$$= -\cos(e^{3x})^2 \cdot 3e^{3x} + \cos(5x)^2 \cdot 5$$

$$= \underbrace{5}_{1}\,\underbrace{\cos 25x^2}_{\frac{1}{2}}\,\underbrace{-}_{1}\,\underbrace{3e^{3x}}_{1}\,\underbrace{\cos e^{6x}}_{\frac{1}{2}}$$

Let $g(x) = \int_{-6}^{x} f(t)\,dt$, where f is the function whose graph is shown on the right. **SCORE: _____ / 7 PTS**

[a] Write "I UNDERSTAND THAT THE GRAPH SHOWS f, BUT THE QUESTIONS ASK ABOUT g".



[b] Find $g'(-1)$. **Explain your answer very briefly.**

$g'(x) = f(x)$

$g'(-1) = f(-1) = 2$

ALL ITEMS ① POINT EACH ON ALL PARTS

[c] Find all intervals over which g is concave up. **Explain your answer very briefly.**

$g'(x) = f(x)$ INCREASING ON $(-6, -1)$

[d] Find all intervals over which g is decreasing. **Explain your answer very briefly.**

$g'(x) = f(x) < 0$ ON $(-6, -3)$ AND $(1, 6)$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{6\cos 2\theta}{1+4\sin^2 2\theta}\, d\theta$$

1 $u = 2\sin 2\theta$

$\theta = \frac{\pi}{2} \longrightarrow u = 0$

$\theta = \frac{\pi}{6} \longrightarrow u = \sqrt{3}$

$du = 4\cos 2\theta \, d\theta$

$\frac{3}{2} du = 6\cos 2\theta \, d\theta$

$\frac{1}{2}$ 1

$$\int_{\sqrt{3}}^{0} \frac{3}{2} \frac{1}{1+u^2}\, du$$

$\frac{1}{2}$ $\frac{3}{2} \tan^{-1} u \Big|_{\sqrt{3}}^{0}$

1

$= \frac{3}{2}\left(\tan^{-1} 0 - \tan^{-1}\sqrt{3}\right)$

$= \frac{3}{2}\left(0 - \frac{\pi}{3}\right)$

$= -\frac{\pi}{2}$ $\frac{1}{2}$

$$\int_{-1}^{1} \frac{4t^3-16t}{t^4-16}\,dt$$

$$\frac{4(-t)^3-16(-t)}{(-t)^4-16} = \frac{-4t^3+16t}{t^4-16}$$

$$= -\frac{4t^3-16t}{t^4-16} \quad 1$$

INTEGRAND IS ODD + CONTINUOUS $\frac{1}{2}$ $\frac{1}{2}$

SO INTEGRAL IS 0

$\frac{1}{2}$

$$\int_{-1}^{1} \frac{\sin y}{1-\tan^2 y}\, dy$$

$1-\tan^2 y = 0$ WHEN $\tan y = \pm 1$

IE. 1 $y = \pm \frac{\pi}{4}$

$\pm \frac{\pi}{4} \in [-1, 1]$

INTEGRAND IS DISCONTINUOUS $\frac{1}{2}$

SO FTC PART 2 DOESN'T APPLY

$\frac{1}{2}$

$$\int \frac{(2x^3 - 3\sqrt{x})^2}{12x^7}\,dx$$

$$= \int \frac{4x^6 - 12x^{\frac{7}{2}} + 9x}{12x^7}\,dx$$

$$= \int \left(\frac{1}{3}x^{-1} - x^{-\frac{7}{2}} + \frac{3}{4}x^{-6}\right)dx \quad 1$$

$$= \frac{1}{3}\ln|x| - \left(-\frac{2}{5}\right)x^{-\frac{5}{2}} + \frac{3}{4}\left(-\frac{1}{5}\right)x^{-5} + C$$

$$= \underbrace{\frac{1}{3}\ln|x|}_{\frac{1}{2}} + \underbrace{\frac{2}{5}x^{-\frac{5}{2}}}_{\frac{1}{2}} - \underbrace{\frac{3}{20}x^{-5}}_{\frac{1}{2}} + \underbrace{C}_{\frac{1}{2}}$$

In manufacturing, marginal cost (measured in dollars per unit) is the cost per unit of producing additional units. **SCORE: _____ / 2 PTS**

At a toy manufacturing plant, if $M(x)$ is the marginal cost when x toys have been produced, what is the meaning of the equation

$$\int_{4000}^{6000} f(x)\,dx = 3000\ ?$$

NOTES: **Your answer must use all three numbers from the equation, along with correct units.**
Your answer should NOT use "x", "f(x)", "integral", "antiderivative", "rate of change" or "derivative".
Your answer should sound like normal spoken English.

IT COSTS \$3000 TOTAL
TO INCREASE THE NUMBER OF TOYS PRODUCED
FROM 4000 TOYS TO 6000 TOYS

GRADED BY ME

In complete sentences, using proper English and mathematical notation, state the Fundamental Theorem of Calculus (both parts) and the Net Change Theorem.

SCORE: _____ / 5 PTS

SEE LECTURE NOTES

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